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Among the fundamental questions in computer science, at least two have a deep impact on mathematics. What can computation compute? How many steps does a computation require to solve an instance of the 3-SAT problem? Our work addresses the first question, by introducing a new model called the *ex-machine* [3]. The ex-machine executes Turing machine instructions and two special types of instructions. *Quantum random instructions* are physically realizable with a quantum random number generator [4, 6]. *Meta instructions* can add new states and add new instructions to the ex-machine.

A countable set of ex-machines is constructed, each with a finite number of states and instructions; each ex-machine can compute a Turing incomputable language, whenever the quantum randomness measurements behave like unbiased Bernoulli trials. In 1936, Alan Turing posed the halting problem for Turing machines and proved that this problem is unsolvable for Turing machines. Consider an enumeration $\mathcal{E}_a(i) = (\mathfrak{M}_i, T_i)$ of all Turing machines $\mathfrak{M}_i$ and initial tapes $T_i$, each containing a finite number of non-blank symbols. Does there exist an ex-machine $\mathfrak{X}$ that has at least one evolutionary path $\mathfrak{X} \to \mathfrak{X}_1 \to \mathfrak{X}_2 \to \ldots \to \mathfrak{X}_m$, so at the $m$th stage ex-machine $\mathfrak{X}_m$ can correctly determine for $0 \leq i \leq m$ whether $\mathfrak{M}_i$'s execution on tape $T_i$ eventually halts? We construct an ex-machine $\mathfrak{Q}(x)$ that has one such evolutionary halting path.

The existence of this path suggests that David Hilbert [5] may not have been misguided to propose that mathematicians search for finite methods to help construct mathematical proofs. Our refinement is that we cannot use a fixed computer program that behaves according to a fixed set of mechanical rules. We must pursue computational methods that exploit randomness and self-modification [1, 2] so that the complexity of the program can increase as it computes.

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